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INTELLECTUAL CONTROL SYSTEMS, DATA ANALYSIS

A Modification of the Multidimensional Fuzzy Controller with the Local Term Division of Input Variables

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Abstract—This paper considers a novel modification of the multidimensional fuzzy controller with interrelated adjustable parameters. The block diagram of the controller and expressions for calculating its main parameters are provided. A transition mechanism from the general term division of input variables to their local one with respect to output variables is described. The authors' special software for the visual design of the fuzzy controller, flexible term adjustment, and the assessment and analysis of the system of production rules is presented. Theoretical results are illustrated by examples.

Keywords: multidimensional fuzzy controller, system of production rules, modification, software, term division

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1. INTRODUCTION

Currently, fuzzy controllers and their modifications, including hybrid ones, are increasingly applied in the development of automated control systems (ACSs). They have more flexible adjustment compared to traditional PID controllers, which is especially important for multidimensional and multivariable systems with a complex mathematical model or without any mathematical model as such. Fuzzy logic is used to design expert and intelligent control systems [1–7].

The practical implementation issues of fuzzy controllers were considered by several authors. In this context, we note the following drawbacks [8–14]:

1) the high complexity of fuzzy control algorithms (including fuzzification and defuzzification);

2) the ambiguous choice of an appropriate membership function of continuous variables;

3) significant (in some cases, multiple) growth of the number of production rules with increasing the number of input variables;

4) no flexibility in the implementation of unconventional approaches to fuzzy controllers, etc.

In addition, third-party software for programmable logic controllers (PLCs), particularly MATLAB [15–17], is often used in the design and program implementation of fuzzy controllers. This requires additional time and funding as well as complicates the introduction of changes during ACS operation.

To settle these issues, it is necessary to improve the characteristics of fuzzy controllers and their operation algorithms; propose new modifications associated with the formation of a system of production rules; and develop special software with the generation of program blocks of fuzzy controllers for modern programming environments for PLCs.

The aim of this paper is to develop, first, a novel modification of the multidimensional fuzzy controller in order to eliminate the above drawbacks and issues and, second, software for adjusting its parameters and the system of production rules.

2. DESCRIPTION OF THE FUZZY CONTROLLER MODIFICATION

We propose the following modification of the fuzzy controller, intended for ACSs with a large number of production rules describing the operation logic of a complex technological object without an adequate mathematical model. The modification is to settle the issues associated with the compilation of a system of production rules (SPRs), eliminate the redundancy of rules, and reduce their maximum number. This is achieved by determining the interrelations of variables and the transition from the general to local term division of input variables.

Figure 1 shows the block diagram of this modification of the multidimensional fuzzy controller with interrelated controlled parameters, with the following notation: $\mathrm{F}_1, \ldots, \mathrm{F}_n$ are the fuzzifiers of input variables x; $\mathrm{D}_1, \ldots, \mathrm{D}_m$ are the defuzzifiers of output variables y; FB and DB are fuzzification and defuzzification blocks, respectively, that control the operation of fuzzifiers and defuzzifiers; LIB is a logical inference block of the controller with a production rule block PRB; T_t are the terms of variables handled by LIB at time t; PrB is a block for predicting the terms of input variables x; FEB are blocks for expanding the controller's functions, including communication and data exchange with external devices.

Let the term set T^x of a continuous variable x have the form

$$T^x = \left\{ T_1^x \left(x_1^{(\mathrm{l})} \leqslant x_t < x_1^{(\mathrm{r})} \right),\ T_2^x \left(x_2^{(\mathrm{l})} \leqslant x_t < x_2^{(\mathrm{r})} \right), \ldots, T_{k(x)}^x \left(x_{k(x)}^{(\mathrm{l})} \leqslant x_t \leqslant x_{k(x)}^{(\mathrm{r})} \right) \right\},$$

where $x^{(\mathrm{l})}$ and $x^{(\mathrm{r})}$ are values determining the left and right limits of the working ranges of the terms T; x_t is the value of the variable x at a time t; $k\,(x)$ is the total number of terms.

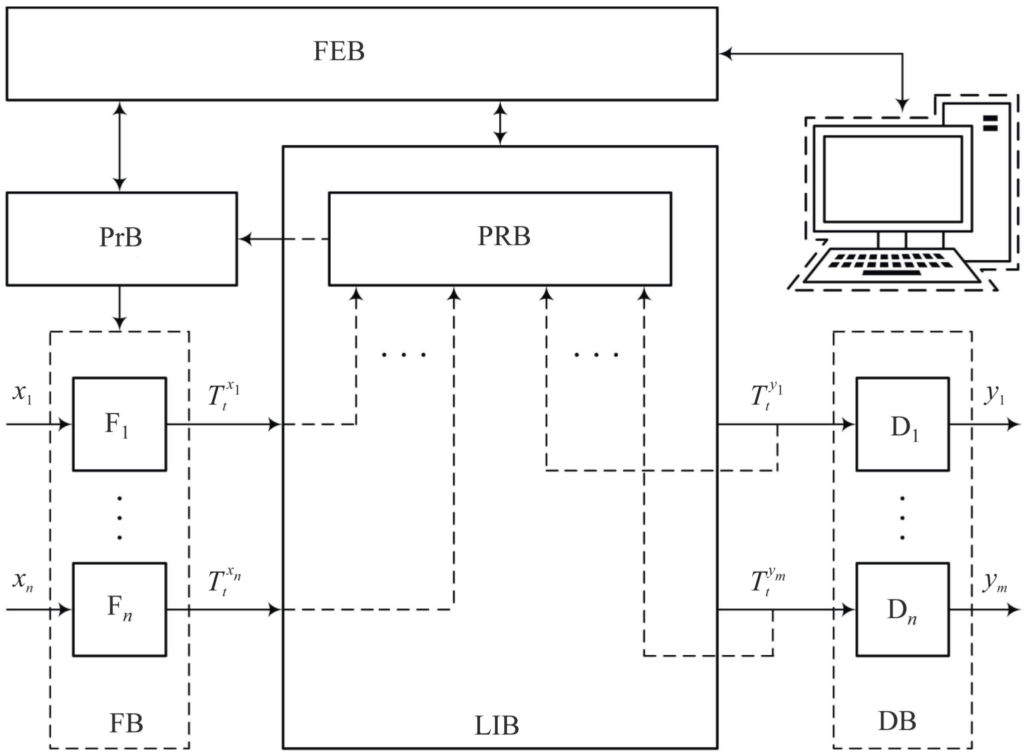

Fig. 1. The block diagram of the fuzzy controller modification.

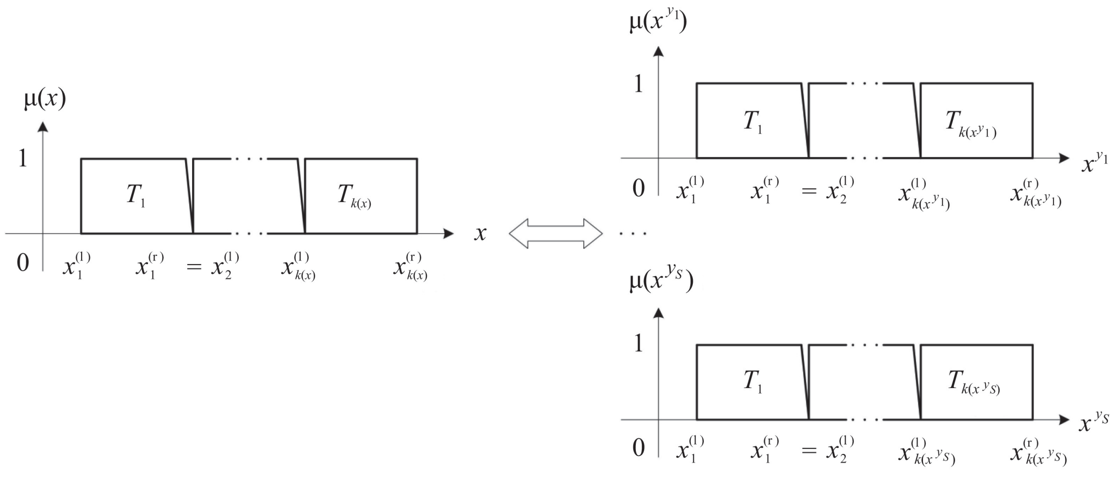

Fig. 2. The division of the general term set into local term sets.

To improve illustrativeness, we choose the class of trapezoidal (close to rectangular) membership functions of x, which is characterized by the empty intersection of the ranges of terms within the general term division of the variable.

By definition,

$$x_k^{(\mathrm{l})} = x_{k-1}^{(\mathrm{r})},$$

and therefore, the value of one range limit is sufficient to describe $k\left(x\right) - 1$ terms T. The limits of the first or last term are described directly by two values. For example,

$$T^x = \left\{T_1^x\left(x_1^{(\mathrm{l})}\right),\ T_2^x\left(x_2^{(\mathrm{l})}\right), \ldots,\ T_{k(x)}^x\left(x_{k(x)}^{(\mathrm{l})} \leqslant x_t \leqslant x_{k(x)}^{(\mathrm{r})}\right)\right\}.$$

As is known, the maximum number $R_{\max}$ of the controller's production rules depends on the total number of the terms of its input variables [18], i.e.,

$$R_{\max} = \prod_{i=1}^{n} k\left(x_i\right),$$

where n specifies the number of the controller's input variables.

In the presence of output variables with feedback (see Fig. 1), the expression for calculating $R_{\max}$ takes the form

$$R_{\max} = \prod_{i=1}^{n} k\left(x_i\right) \prod_{j=1}^{o} k\left(y_j\right),$$

where o is the number of output variables of the feedback controller.

For a multidimensional fuzzy controller with m output variables y, the general term division of some input variables x may be redundant for calculating the values of separate output variables and cause defuzzification errors associated primarily with the absence of production rules defined by technical specifications.

The general term division can be simplified to a local one with respect to particular variables y. For instance, the local term set T^{x^y} of a continuous variable x with respect to an interrelated variable y can be analytically represented as follows:

$$T^{x^y} = \left\{T_1^{x^y}\left(x_1^{y(\mathrm{l})} \leqslant x_t < x_1^{y(\mathrm{r})}\right),\ \ldots,\ T_{k(x^y)}^{x^y}\left(x_{k(x^y)}^{y(\mathrm{l})} \leqslant x_t \leqslant x_{k(x^y)}^{y(\mathrm{r})}\right)\right\},$$

where $k\left(x^y\right) \leqslant k\left(x\right)$.

This process is visualized in Fig. 2.

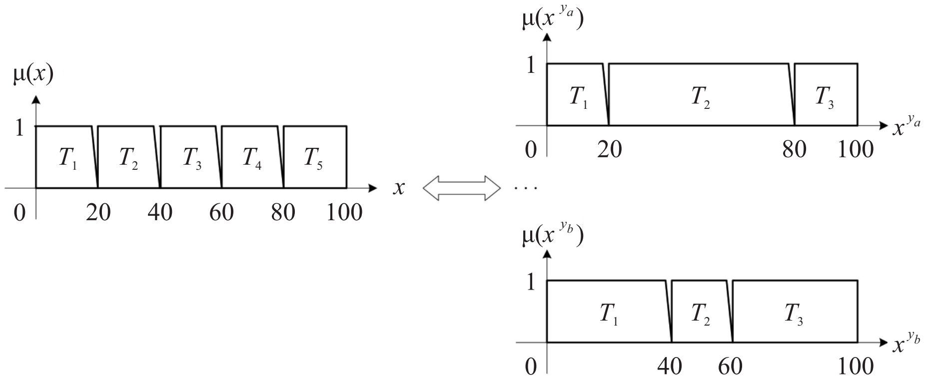

Fig. 3. The conflict of local terms when interpreting variables.

In this case, the expression for calculating the maximum number $R'_{\max}$ of the controller's production rules takes the form

$$R'_{\max} = \sum_{j=1}^{m} R_{\max}^{y_j} = \sum_{j=1}^{m} \prod_{i=1}^{s} k\left(x_i^{y_j}\right),$$

where $R_{\max}^{y}$ is the maximum number of production rules for a particular output variable y; s denotes the number of input variables x interrelated with the output variable y.

To perform calculations, for the sake of convenience, we present the initial data on the number of the terms of input variables x for their interrelated output variables y in Table 1. Here, the symbol "–" in particular cells indicates the absence of interrelation between the corresponding variables.

Table 1. Data to analyze the term relations of variables

Output variable	The number of the terms of input variables		
	x_1	...	x_n
y_1	$k\left(x_1^{y_1}\right)$	...	$k\left(x_n^{y_1}\right)$
...	...	...	...
y_m	$k\left(x_1^{y_m}\right)$	...	$k\left(x_n^{y_m}\right)$

In this case, the minimum number of the terms of output variables must satisfy the condition

$$k_{\min}(y) \leqslant R^y,$$

where R^y is the number of production rules for a particular output variable y.

If the number R^y of production rules is less than the total number of the terms of y, some terms will remain unused (not participate in forming the system of production rules). Such terms should be removed by uniting with neighbor ones to avoid gaps within the working ranges of the output variables. If the union of terms is impossible, such variables cannot be used for feedback implementation in order to avoid fuzzification errors.

This approach allows eliminating the so-called conflict of local terms due to the overlap of their ranges when forming the general term division (Fig. 3), which leads to the emergence of new terms and consequently increases the maximum number of production rules of the controller.

According to Fig. 3, the general term division of a continuous variable x,

$$T^x = \{T_1^x(0),\ T_2^x(20),\ T_3^x(40),\ T_4^x(60),\ T_5^x(80 \leqslant x_t \leqslant 100)\},$$

is formed using the following local term divisions of x with respect to the output variables y_a and y_b :

$$T^{x^{y_a}} = \left\{T_1^{x^{y_a}}(0),\ T_2^{x^{y_a}}(20),\ T_3^{x^{y_a}}(80 \leqslant x_t \leqslant 100)\right\},$$
$$T^{x^{y_b}} = \left\{T_1^{x^{y_b}}(0),\ T_2^{x^{yb}}(40),\ T_3^{x^{y_b}}(60 \leqslant x_t \leqslant 100)\right\}.$$

For instance, the term T_1^x corresponds to the term $T_1^{x^{y_a}}$; the term T_3^x, to the term $T_2^{x^{y_b}}$; the term T_5^x, to the term $T_3^{x^{y_a}}$. The terms T_2^x and T_4^x are obtained by uniting the ranges of the remaining terms from the local term divisions of the continuous variable x with respect to the output variables y_a and y_b.

In this example, the general term division of the continuous variable x leads to the following problems. The terms T_1^x and T_2^x, T_4^x and T_5^x duplicate the production rules for the output variable y_b, and the terms T_2^x, T_3^x, and T_4^x do the same for the output variable y_a. This affects the total number of production rules, especially when developing multivariable fuzzy systems with

$$y = f(x_1, \ldots x_s).$$

Moreover, uniting terms can lead to the incorrect operation of defuzzifiers with certain classes of membership functions (e.g., triangular) and several difficulties when forming a system of production rules.

Local term division resolves these problems, allowing the engineer to develop a fuzzy ACS in full compliance with technical specifications.

If necessary, critical production rules can be identified: they involve the terms of continuous variables marked as “critical” to indicate their particular importance (e.g., when determining the extreme ranges of values as an emergency signal).

The number of critical rules for each group of interrelated variables, $R^g_{\text{gr cr}}$, can be calculated as follows:

$$R^g_{\text{gr cr}} = R^g_{\text{gr}} - \prod_{i=1}^{n_g} \left(k\left(x_i\right) - k_{\text{cr}}\left(x_i\right)\right),$$

where k_{cr} is the number of critical terms defined for input variables x in the volume n_g, forming a group with the ordinal number g; R^g_{gr} is the maximum number of production rules of the group:

$$R^g_{\text{gr}} = \prod_{i=1}^{n_g} k\left(x_i\right).$$

3. DESCRIPTION OF THE CONTROLLER SOFTWARE

Modern design and development environments for fuzzy controllers have no capabilities for determining the interrelations of variables and passing from the general term division of input variables to local ones with respect to output variables, as well as no deep analysis tools for a system of production rules. Therefore, we developed a software solution (computer-aided system) for the visual development of a PLC program for the above modification of the multidimensional fuzzy controller, the calculation and analysis of its main parameters, the formation of production rules, etc.; see Figs. 4 and 5.

When creating a new fuzzy controller project, the following basic information is entered: the number of input and output variables, their working ranges, and interrelations. Terms are modified, and production rules are entered, in a special visual editor.

After data entry is complete, the user can analyze the term divisions of variables and the system of production rules. In this case, the total number of terms and the number of critical terms (for the entire controller and each variable separately), the belonging of variables to a particular group (control loop), etc., are assessed.

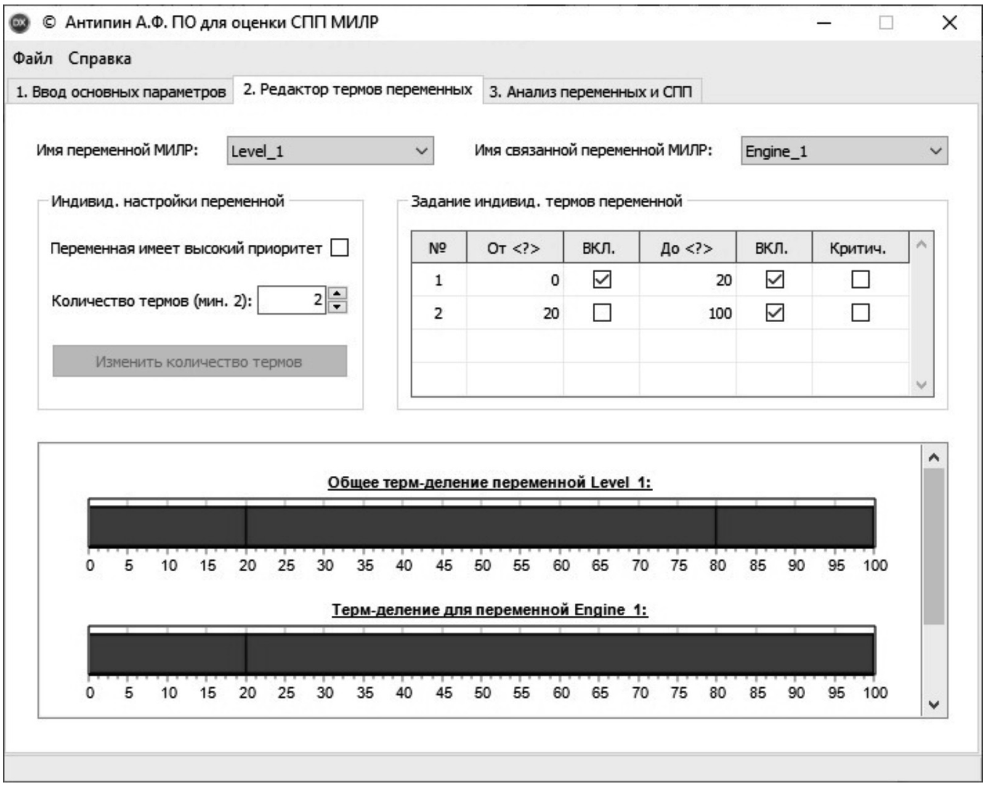

Fig. 4. The term editor for continuous variables and their interrelations.

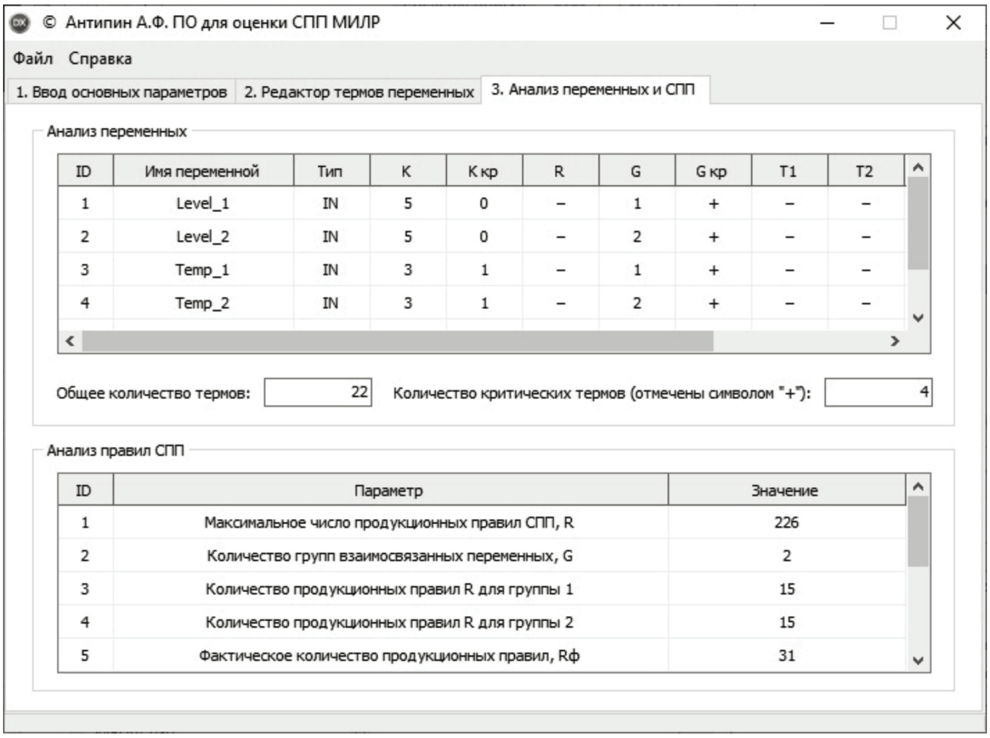

Fig. 5. Analysis results for variables and the system of production rules.

```
VAR_INPUT
    NUM: INT;
    . . .
    OUT_RM: ARRAY [0..R-1] OF BYTE;
END_VAR

VAR_OUTPUT
    . . .
    OUT_M: REAL;
END_VAR

VAR
    . . .
    OUT_MM: ARRAY [0..K(OUT_M)] OF REAL;
END_VAR
. . .

// DEFUZZIFIER M
IF BYTE_TO_INT(OUT_RM[NUM]) > 0
    THEN OUT_M := (OUT_MM[BYTE_TO_INT(OUT_RM[NUM]) –1]
                  + OUT_MM[BYTE_TO_INT(OUT_RM[NUM])]) / 2;
END_IF;
```

Fig. 6. SCL program code of the defuzzification functional block.

A separate table displays the values of the controller parameters depending on the content of the system of production rules (Fig. 5):

1) the maximum $R_{\max}$ and factual $R'_{\max}$ numbers of production rules forming the system of production rules of the controller;

2) the number of groups of interrelated variables;

3) the number R^g_{gr} of production rules for each group, the number $R^g_{\text{gr cr}}$ of critical rules, etc.

The settings of the interrelations and term divisions of variables can be placed in a separate file of a special format and loaded into the computer-aided design system developed by the authors [19]. This system contains an editor for the system of production rules and special tools for checking its completeness as well as the correctness of the user data.

Upon completion of the work, it is possible to generate a ready-made program of the controller and/or its separate functional blocks (fuzzifiers and defuzzifiers) in the SCL and AWL format for Simatic S7 PLCs considering the requirements of the IEC 61131-7:2000 standard (Programmable controllers – Part 7: Fuzzy control programming).

Figure 6 presents a fragment of the generated SCL program code of the defuzzification functional block for a continuous output variable M whose crisp value is found by calculating the midpoint of the range of the term active at the current time [18].

4. COMPUTATIONAL EXPERIMENT

We carry out a computational experiment to assess the system of production rules of the fuzzy controller when passing from the general to local term division of input variables.

Consider a control loop with four input and two output continuous variables x and y, respectively; their interrelation diagram and general term divisions in the working range [0, 100] (in simplified form) are shown in Fig. 7. Since the terms of the output quantities do not affect the total number of production rules, they can be omitted for calculations.

The term sets of the input continuous variables have the form

$$T^{x_1} = \{T_1(0),\ T_2(10),\ T_3(25),\ T_4(50),\ T_5(75), T_6(90;100)\},$$
$$T^{x_2} = \{T_1(0),\ T_2(25),\ T_3(75;100)\},$$
$$T^{x_3} = \{T_1(0),\ T_2(10),\ T_3(20),\ T_4(50),\ T_5(80), T_6(90;100)\},$$
$$T^{x_4} = \{T_1(0),\ T_2(15),\ T_3(85;100)\}.$$

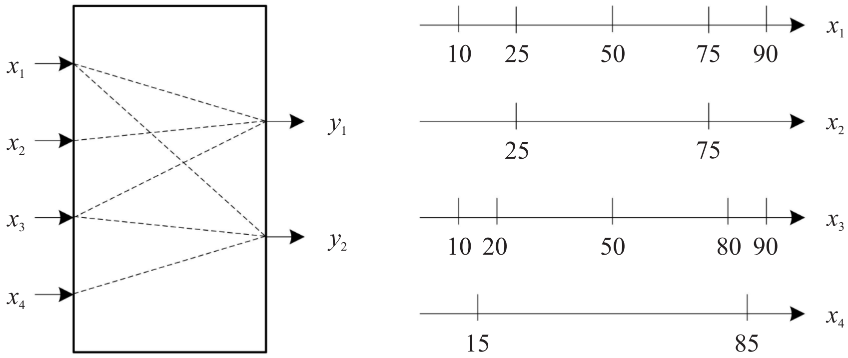

Fig. 7. The interrelation diagram and general term division of input variables.

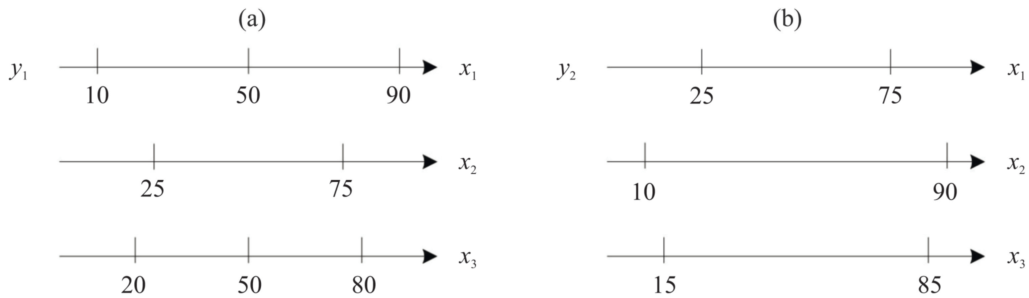

Fig. 8. The local term division of the input variables x_1, x_2, x_3 and x_4 with respect to the output variables (a) y_1 and (b) y_2.

Thus, $k(x_1) = 6$, $k(x_2) = 3$, $k(x_3) = 6$, and $k(x_4) = 3$. The maximum number $R_{\max}$ of production rules is

$$R_{\max} = \prod_{i=1}^{4} k(x_i) = 324.$$

Being extremely redundant, this number will strongly complicate the formation of the system of production rules of the controller. However, certain interrelations can be used at the controller design stage to pass from the general to local term division, and production rules can be subsequently introduced for each output variable separately.

As is known, the input variables x_1, x_2, and x_3 are related to the output variable y_1 whereas x_1, x_3, and x_4 to y_2. Figure 8 demonstrates their local term division.

The term sets of the continuous input variables x_1, x_2, and x_3 with respect to the continuous output variable y_1 have the form

$$T^{x_1^{y_1}} = \{T_1(0),\ T_2(10),\ T_3(50),\ T_4(90;100)\},$$
$$T^{x_2^{y_1}} = \{T_1(0),\ T_2(25),\ T_3(75;100)\},$$
$$T^{x_3^{y_1}} = \{T_1(0),\ T_2(20),\ T_3(50),\ T_4(80;100)\}.$$

Next, the term sets of the continuous input variables x_1, x_3, and x_4 with respect to the continuous output variable y_2 have the form

$$T^{x_1^{y_2}} = \{T_1(0),\ T_2(25),\ T_3(75;100)\},$$
$$T^{x_3^{y_2}} = \{T_1(0),\ T_2(10),\ T_3(90;100)\},$$
$$T^{x_4^{y_2}} = \{T_1(0),\ T_2(15),\ T_3(85;100)\}.$$

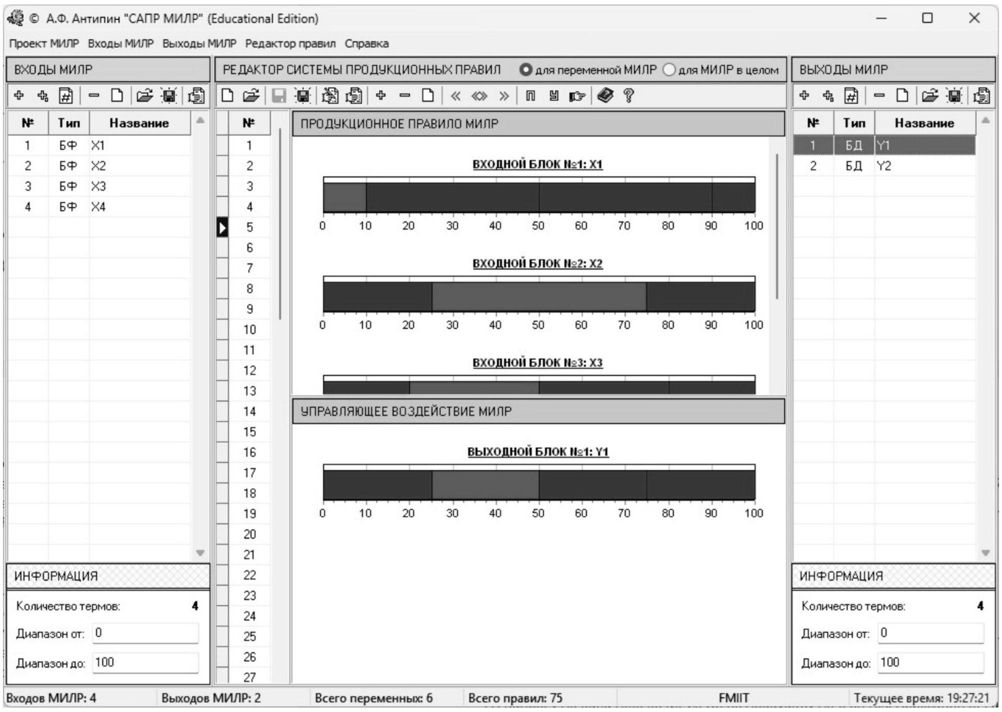

Fig. 9. The editor of the computer-aided design system.

The numbers of the terms of the input variables x for particular output variables y are combined in Table 2.

Table 2. Data to analyze the term relations of variables

Output variable	The number of terms of input variables			
	x_1	x_2	x_3	x_4
y_1	4	3	4	–
y_2	3	–	3	3

Thus, in view of the known interrelations between the input and output variables, the maximum number $R'_{\max}$ of production rules is

$$R'_{\max} = \sum_{j=1}^{2} R_{\max}^{y_j} = k\left(x_1\right) k\left(x_2\right) k\left(x_3\right) + k\left(x_1\right) k\left(x_3\right) k\left(x_4\right) = 75.$$

The resulting reduction in the maximum number of production rules, $\Delta R_{\max}$, reaches

$$\Delta R_{\max} = \frac{R_{\max} - R'_{\max}}{R_{\max}} = 0.8$$

(4.3 times).

This process can be automated using the above software solution. Based on the calculation results, it is possible to assess the design complexity and save the settings of the interrelations and term division of the variables for subsequent SPR formation in the computer-aided design system (Fig. 9).

Working in a single visual programming environment simplifies coding for the multidimensional fuzzy controller with interrelated adjustable parameters as well as calculating its basic characteristics.

5. CONCLUSIONS

The novel modification of the multidimensional multivariable fuzzy controller described in this paper provides several advantages in automated control system design: the flexible adjustment of the terms of variables, reduction in the maximum number of production rules, a separate defuzzification function specified for each output variable [20], etc.

The original special software developed by the authors contains tools for the visual design of the fuzzy controller, the evaluation and analysis of the system of production rules, and the generation of a ready program for the Simatic S7 PLC. This software solution can be used by experts and engineers to design automated control systems for complex technological objects without adequate mathematical models and create reliable and effective PLC programs for fuzzy controllers, reducing the time of their development.

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